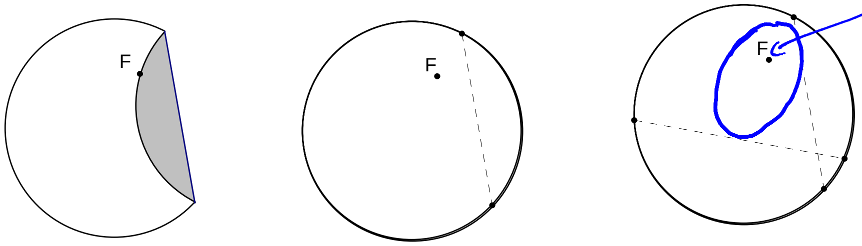
Folding Paper from Geometry through Calculus:
Scott Galson & P.J. Karafiol
Friday, January 23, 2009
11:10 AM

1. Folding a circle (G & PC) (SG)

Plot a point somewhere inside your circle and label it F.
Fold your circle so that a point on the edge lands on F.



not the center
Maybe a circle?
maybe an ellipse?
Is the other focus at the center?

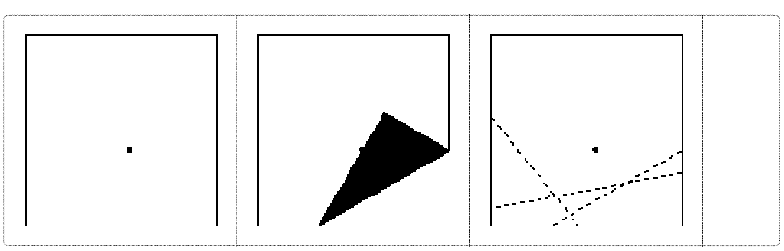
Unfold the circle, and repeat so that a different point on the circle lands on F.

Repeat many times (when you get really bored, about ten more times should do it).

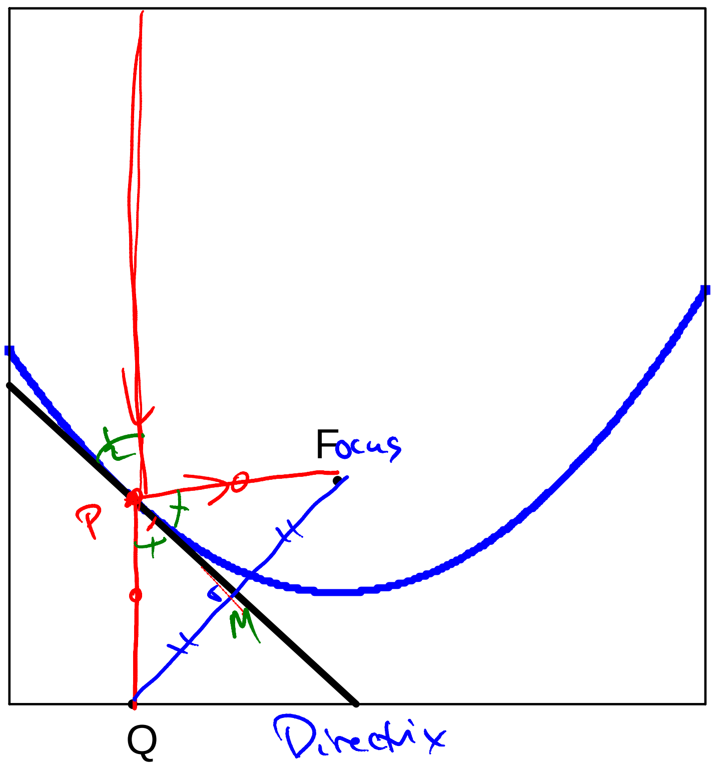
What shape do you see?

2. A simpler case: folding a square (G & PC) (SG-PJK)

Repeat the process with a square: plot a point F, fold the bottom edge of the square onto the point, unfold, and repeat, <u>using the bottom edge each time!!!</u>



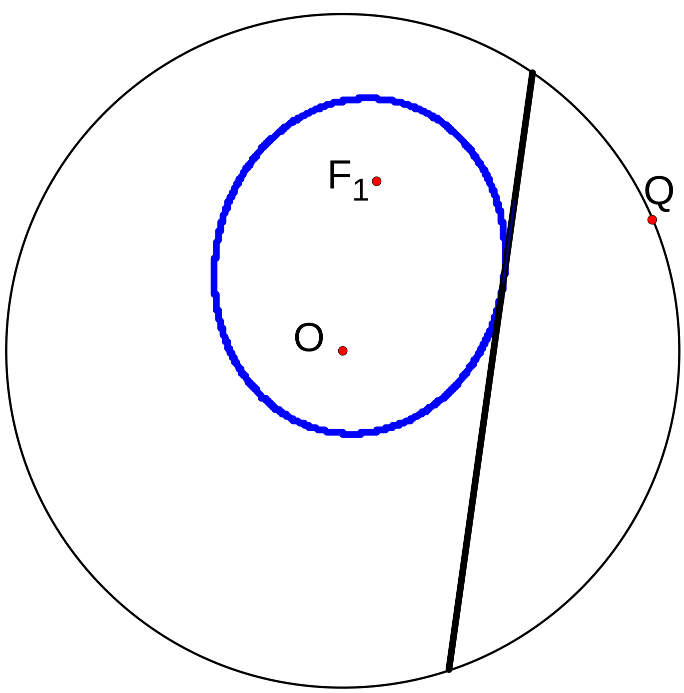
What shape do you see? Why?

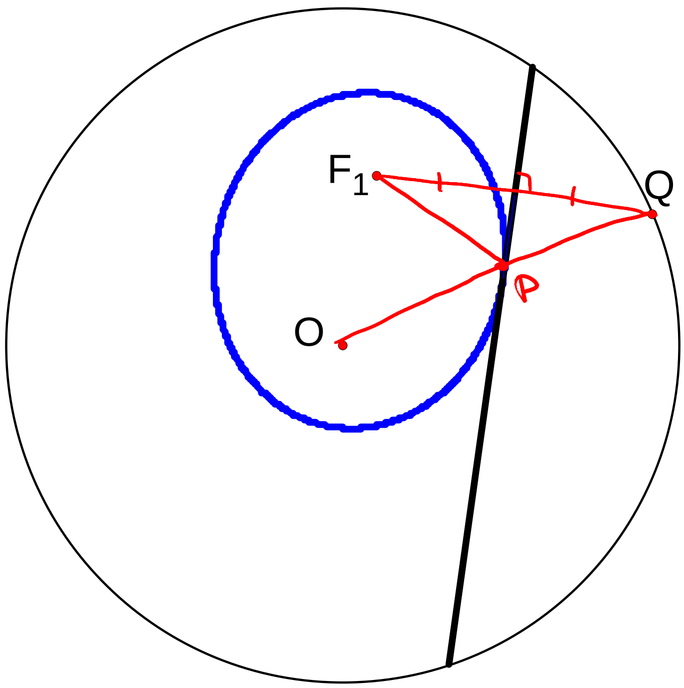


$\angle QPM \cong \angle MPF$

$\angle QPM \cong \angle 2$

3. Folding a circle: reprise (G & PC) (PJK)



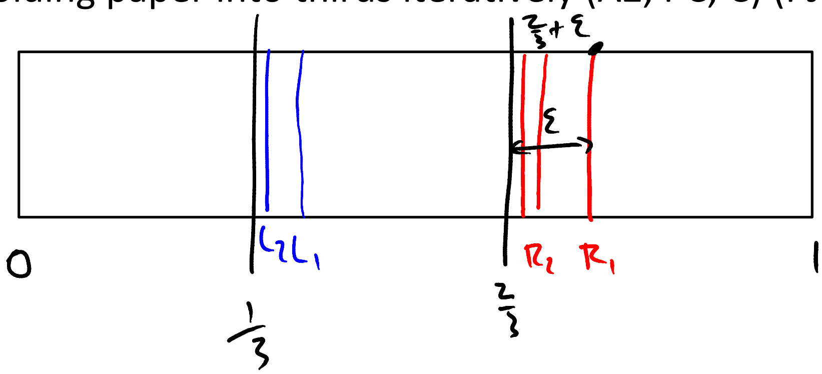


As Q moves around the ⊙, OQ is invariant

$F_1P = PQ$, so

$F_1P + OP = OP + PQ$
= constant

4. Folding paper into thirds iteratively (A2, PC, C) (PJK)



$R_1 = \frac{2}{3} + \varepsilon$

$L_1 = \frac{0 + \frac{2}{3} + \varepsilon}{2}$

$= \frac{1}{3} + \frac{\varepsilon}{2}$

$R_2 =$ MP of $\overline{L_1 1}$

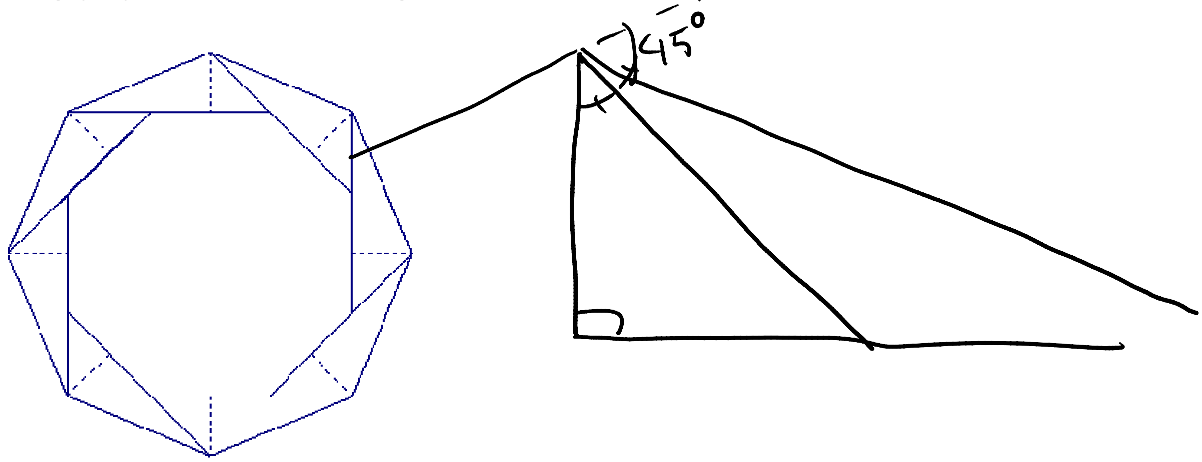
$= \frac{1 + \frac{1}{3} + \frac{\varepsilon}{2}}{2}$

$= \frac{2}{3} + \frac{\varepsilon}{4}$

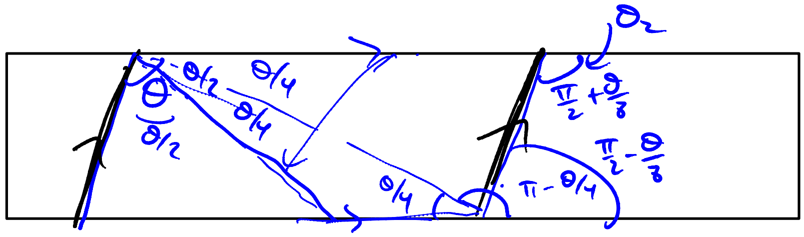
Optimistic Algorithm

=

5. Folding paper into an octagon (G) (SG)



6. Investigating folding sequences (G-C?) (PJK)



$$\theta = \frac{\pi}{2} + \frac{\theta}{8}$$

$$\frac{7\theta}{8} = \frac{\pi}{2}$$

$$\theta = \frac{4\pi}{7}$$

① If original ∠ was $\theta_1 = \frac{4\pi}{7} + \varepsilon$

Then $\theta_2 = \frac{4\pi}{7} + \frac{\varepsilon}{8}$?

② What are the lengths of the creases?

③ What sequence of Downs & Ups gives me a particular angle?

④ Can any particular angle be the limit of an iterated folding sequence? (exterior ∠s of regular polygons?)

sites.wpcp.org/karafiolp