One Problem, Three Approaches

On November 1, 100 people at Payton (total population: 1000) report a strange illness that makes them crave tater tots. On November 2, the number of infected people is 120. What should we expect?

Model 1: Constant Rate of Change

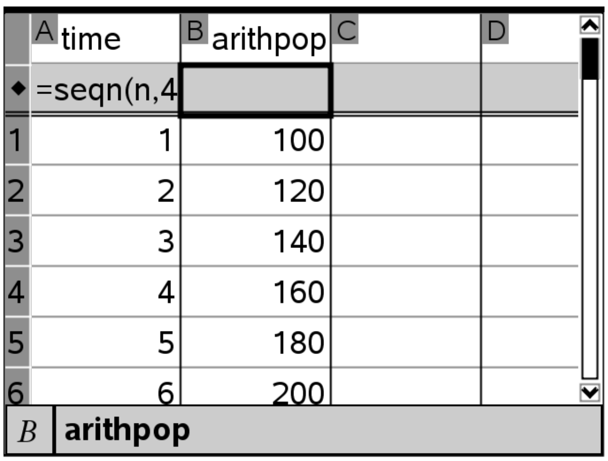
Suppose we assume that *every* day 20 more people will get the disease. Let a_n be the number of people with the disease on day n, with $a_1 = 100$.

a. Write the number of people who have the disease on day n as a recursive formula: ______________

b. What kind of sequence is $\{a_n\}$? ______________________

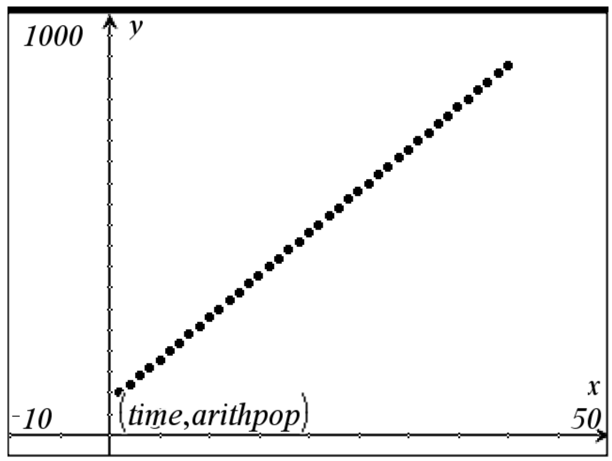
c. On your calculator, create a new problem, and create a spreadsheet page. Label column A **Time** and fill cells A1-A40 with the numbers 1,2,3,….

d. Now label column B **Arithpop**. In B1 put the value 100; in B2 put the formula **`=b1+20`**. Then fill down to cell B40. Your spreadsheet should look like the one below left.



	A time	B arithpop	C	D
◆	=seqn(n,4			
1	1	100		
2	2	120		
3	3	140		
4	4	160		
5	5	180		
6	6	200		

B arithpop



e. Get a graphs and geometry page, and from the **Graph Type** menu choose **Scatterplot**. In the *x*-list select **time**, and in the y-list select **arithpop.** Set an appropriate window. Your graph should look like the one above right.

Think for a minute:

- ✎ What are some reasons why this model is plausible?

- ✎ What kind of *continuous* function does this model yield?

- ✎ What are some of its defects?

Model 2: Constant Rate of Growth

What if we assume that the infection spreads at the same *percentage rate* each day? One reason for this assumption: the number of people who get infected depends on the number of people who already have the disease, because people catch the disease from people who already have it. Again, $a_1 = 100$; $a_2 = 120$.

a. Write the number of people who have the disease on day n as a <u>recursive</u> formula: ______________

b. What kind of sequence is $\{a_n\}$? ______________________

c. Go back to your spreadsheet page and label column C **Geopop**. In C1 put the value 100; in C2 put the formula corresponding to your recursive formula in Part (a). Then fill down to cell C40.

d. Add **Geopop** to your previous graph by going back to the graph and geometry page, tabbing down to the stat plot list at the bottom, and going down to s2.

✎ What are some differences between the Geopop and Arithpop models?

✎ Which features of the Geopop model are *more* plausible than the Arithpop model?

✎ Which ones are less?

Model 3: Logistic Growth

What if we assume that rate of infection depends *both* on the number of people who have the disease *and* on the number of uninfected people? Again, $a_1 = 100$ and $a_2 = 120$.

a. The recursive formula is of the form $a_n = a_{n-1} + r \bullet a_{n-1} \bullet \left(1 - \frac{a_{n-1}}{P}\right)$, where P is the total population. Compute the value of r in this case by substituting known values for the other variables. (Payton's total population is 1000, including adults.)

b. Go back to your spreadsheet page and label column D **Logpop**. In D1 put the value 100; in D2 put the formula corresponding to your recursive formula in Part (a). Then fill down to cell D40.

d. Add **Logpop** to your previous graph by going back to the graph and geometry page, tabbing down to the stat plot list at the bottom, and going down to s2.

- What are some differences between the Logpop and other models?

- Which features of the Logpop model are *more* plausible than the Geopop model?

- Which features of the Logpop model are *more* plausible than the Arithpop model?

- You can find an explicit formula as follows: From the spreadsheet page, choose the Stats menu, and choose **Logistic Regression (d=0).** Write your formula below.

- Use limit logic to find the horizontal asymptotes of your model explicitly.