Precalculus BC Semester 2 Review Packet

What's due when?

Date	Problems
5/26	11-25 from this sheet, + HW handout
5/27-28	26-45
5/29	46-55 and textbook problems 10.3A #s 1,3,5,7,11, 13 Short quiz on Matrices and Markov day.
6/1	56-66 + 12R 19,23,25,27,43 Hofstadter Extra Credit (if you're doing it)

11. Consider the parabola $(y - 2)^2 = ¼ (x + 3)$. Find the vertex, focus, and directrix of the parabola, and graph it.

12. In ΔABC, $a = 5$ cm, $b = 8$ cm, and $c = 12$ cm. To the nearest 0.1°, find the angles of ΔABC.

13. Find all real numbers x (exactly) for which $2 \sin (5x + \pi/9) = -1$. Don't use your calculator.

14. Find the domain and range of the function $f(x) = 4^{\tan^{-1} x}$.

15a. Find the rectangular coordinates of the point whose polar coordinates are $(6,11\pi/3)$ *without a calculator*.

15b. Use the conversion formulas from 6a to express the equation $4x + 3y = 12$ in polar coordinates.

16. Simplify as much as possible: $(\cos 3x + \sin 5x)^2 - (\sin 3x + \cos 5x)^2$

17. Find a simplified formula for $\sin(\tan^{-1} (4x + 1))$.

18. If the domain of f is restricted to the open interval $(-\pi/2, \pi/2)$, then the range of $f(x) = e^{\tan x}$ is ?

19. What kind of symmetry does the graph of $x^2 + 4xy + 2y^2 - 16 = 0$ have?

19b. Graph it and sketch its graph

20. Standing at Payton, you measure the angle of elevation to two different windows on the Sears Tower. The angle to the lower window is 4.5° and the angle to the upper window is 9.2°. If the two windows are about 500 feet apart, how far away is the Sears Tower?

DRAW A LABELED DIAGRAM and then solve the problem.

21. Let $g(x) = |2\cos x - 1|$, and let (x,y) be the coordinates of the highest point on the graph of $g(x)$. Then (x,y) = ?
→ How could you approach this without a calculator?

22. Using the binomial theorem, find the real part of $(\cos \theta + i \sin \theta)^3$

23. Use your result in 13 to find a formula for $\cos 3\theta$ in terms of $\cos \theta$ and $\sin \theta$.

24. What is the domain of $\cos^{-1} (\ln x)$?

25. Prove that the sum of the squares of the diagonals of a parallelogram equals the sum of the squares of (all four) of its sides. Hint: label the sides & angles with variables & use trig.

26. Solve for x exactly in $[0,2\pi)$, preferably with no calculator: $\sin 3x \cos 2x + \cos 2x \sin 3x = 1$.

27. Simplify $\sin 12° \cos 102° - \cos 12° \sin 102°$ without a calculator.

28. Find the magnitude and direction angle (nearest 0.01°) of the vector -4**i** + -3**j**.

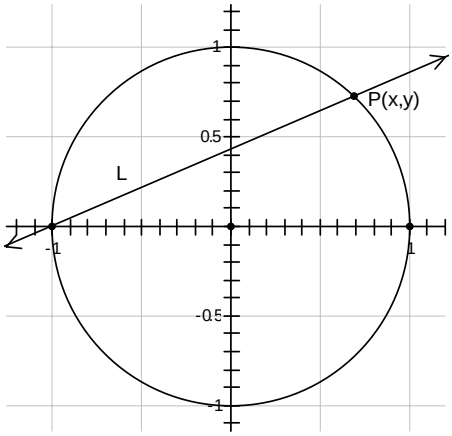
29. If **u** = <3,–5> and **v** = <10,6>, find

a. $\mathbf{u} \cdot \mathbf{v}$ b. $\mathbf{u} \cdot \mathbf{u}$ c. A unit vector in the direction of **u**. d. $\text{comp}_{\mathbf{v}}\,\mathbf{u}$

30. Use DeMoivre's Theorem to find all complex solutions: $x^3 = 1 + i$.

31. If **u** and **v** are nonzero vectors, and $k = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\| \mathbf{v} \|^2}$, show that $\mathbf{u} - k\mathbf{v}$ is orthogonal to **v**.

32. Let *L* be a line through (-1,0) with slope *m*, and let *P* = (*x*,*y*) be the other intersection point of line *L* with the unit circle.



a. Find the equation of line *L*.

b. Find the coordinates of point *P* in terms of *m*.

33. A stone arch is shaped like a parabola with a height of 10 feet in the center and a width of 20 feet. How high is the arch 5 feet from the center?

34. Same as #33, except it's a semiellipse.

35. Find the equation of an ellipse whose major axis endpoints are (4,7) and (4,-3) and which contains the point (6,3).

36. If $\tan(x - y) = 1$ and $\tan x = \frac{1}{2}$, find $\tan y$ exactly.

37. If **v** = 4**i** + 7**j** and **w** = -2**i** + 6**j**, find

a. $\| \mathbf{v} \|$ b. $\mathbf{v} \cdot \mathbf{w}$ c. θ, the angle between **v** and **w**.

38. Prove that for all vectors **v** and real numbers *r*, $\mathbf{v} \cdot r\mathbf{v} = r \| \mathbf{v} \|^2$.

39. If $a = 12$ cm, $b = 18$ cm, and $m\angle A = 20°$, find all possible solutions to ΔABC.

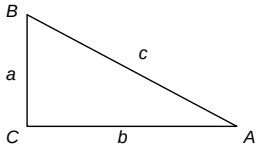
40. Describe the graph of $y = 4 + 8 \sin(\pi x/4 + \pi/8)$ – range, amplitude, period, phase shift.

41a. Is the sequence $\{a_n\}$ whose formula is $a_n = \sqrt{n+1} - \sqrt{n}$ arithmetic, geometric, or neither?

41b. Estimate, but use algebra to justify, $\lim_{n \to \infty} a_n$.

42. PODASIP: $\tan^2 x = \dfrac{1 - \cos 2x}{1 + \cos 2x}$

43. For any right triangle *ABC* with right angle *C*, express $\sin 2\theta$ in terms of *a*, *b* and *c*.



44. If *a* and *b* are in the second quadrant, and $\sin a = 4/5$, $\sin b = 5/13$, find the exact value of $\sin(b - a)$.

45. Express the quotient $\dfrac{1}{1+i}$ in polar form.

46. A force of 30 pounds and a force of 24 pounds are applied on an object at 60° angles to each other.

a. Draw a picture. b. What single force would exactly balance the forces?

47. Pluto travels in an elliptical orbit with the sun at one focus. At its nearest, Pluto is 30 AU's ("Astronomical Units"—1 AU = 93 million miles) from the sun; at the furthest, Pluto is 50 AU's from the sun.

a. Draw the situation and find the ellipse's major axis. b. Now find the ellipse's minor axis and its equation.

48. Blades of a windmill are turning at 50 feet per second under optimum conditions. If the optimum angular velocity is $\pi/2$ radians per second, how long are the blades (to the nearest 0.01 ft?).

49. If $\{a_n\}$ is a geometric sequence and for all n, $b_n = \ln a_n$, what kind of sequence is b_n?

a. Try at least one example. b. Justify a general answer.

50. Consider the curve whose parametric equations are $y = t^3 - 4t$ and $x = 11 - 3t$

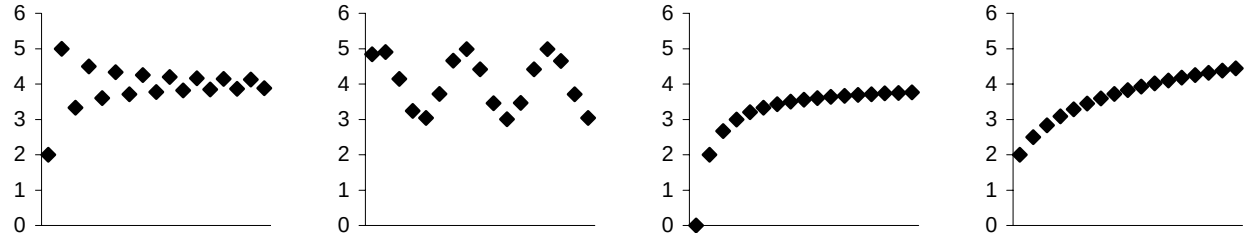
a. Find dx/dt at $t = 4$. b. Find the exact values of the x- and y-intercepts of the curve.

51a. If $f(x) = x^3 - 12x$, find $Df(x)$ 51b. Find $\lim_{h\to 0} Df(x)$, and evaluate your answer at $x = 3$

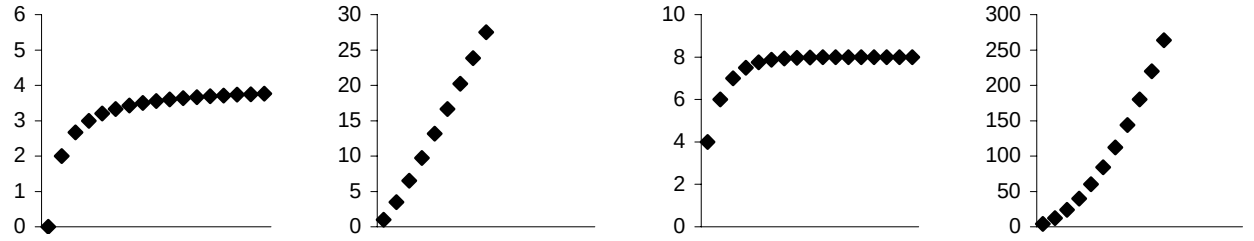
52. What does your answer to 52b tell you about the graph of $f(x)$ at $x = 3$?

53. Jessica leaves WPCP in her car at v miles per hour. A short time t later, John jumps in his car, and tries to catch her by driving at w miles per hour. Let C be the time it takes him to catch up. Write a formula for C in terms of t, v, and w. (You may assume that t is in hours.)

54. If $\lim_{n\to\infty} a_n = 4$, which is/are NOT a plausible graph for the sequence $\{ a_n \}$?



55. If $\lim_{n\to\infty} a_n = 4$, and S_n is the n^{th} partial sum $a_1 + a_2 + \ldots + a_n$, then a plausible graph for S_n is…



56. Find a set of parametric equations for…

a. The line between (1,4,6) and (3,-2,18) b. The circle centered at (-4,4) passing through (3,2)

57. Find all intersection points of the parabola with vertex (0,1) and focus (0,0) and the ellipse centered at the origin with major axis endpoints (±5,0) and foci (±3,0).

58. The graph of $r = 4 \sin 4\theta$ contains several "petals". The values of θ that produce a single petal are in [0,___]?

59a. If $\sin a = 4/15$ and $\pi/2 < a < \pi$, then $\cot a = ?$ b. Find the period of $g(x) = -18\tan(\frac{2\pi}{11}x)$.

60a. Compute $5 + 5.2 + 5.4 + \ldots + 9$ 60b. Compute $5 + \frac{10}{3} + \frac{20}{9} + \ldots$

61. If $a - \frac{3}{5}a + \frac{9}{25}a - \frac{27}{125}a + \ldots = 10$, then $a = ?$

62. If $\tan y = x/b$ and $\tan z = \frac{10 + x}{b}$, solve for y in terms of x and z.

63. If $f(x) = 2^{-x} \sin x$, then $\lim_{x \to \infty} f(x) = ?$ Explain algebraically.

64. A road rises 120 feet per horizontal mile. What is the angle of elevation of the road?

65. PODASIP: $\tan^2 \theta + \sin^2 \theta = (\sec \theta + \cos \theta)(\sec \theta - \cos \theta)$.

66. For what values of t in $[0,2\pi)$ will the geometric series $1 + \frac{\sec t}{2} + \frac{\sec^2 t}{4} + \ldots$ reach a finite number??