It's not really a bear, but a raccoon

Your group will work on one of the four families of polar graphs desribed below, in order to understand family similarities, and differences within the family, as clearly as possible. At the end of class, you'll make a two-minute presentation on your family.

- First, investigate on your Nspire using a slider to investigate similarities and differences. Unless otherwise stated, the domain of the parameter is **R**, so be sure to include noninteger and negative values!
- Some particular categories to think about:
    - Does the graph intersect itself?
    - As θ goes from 0 to 2π, does the graph make one loop or multiple loops?
    - Does the graph cross the axes? For what *r*-values?
    - Does the graph cross the origin? For what θ-values?
- Then, use algebra to explain the results you've discovered and give general answers in terms of the parameters.

Tip: Before you're totally done with a function, try expanding the domain (by dragging the arrows at the end of the plot, or right-clicking and selecting its properties) so that you're sure you see all of it.

Family I: Limaçons and Cardiods

General equation: $r = a + b\cos\theta$,

→ The case $a = b$ is called a *cardiod* – why?

Family 2: Lemniscates

General equation: $r^2 = a^2\cos(n\theta)$, $n \in \mathbf{Z}$ ($a \in \mathbf{R}$)

→ Make sure to plot both positive and negative r's.

Family 3: General Rose Curves

General equation: $r = 1 + a\cos(n\theta)$, $n \in \mathbf{Z}$ ($a \in \mathbf{R}$)

Family 4: Oddly Familiar #2

General equation: $r = \dfrac{ke}{1 - e\cos\theta}$, $k, e \in \mathbf{R}$ (e isn't Euler's number, but a suggestively-named parameter).

Historical note: Limaçons were first discovered by the Dutch mathematician and artist Albrecht Dürer, but they were rediscovered and their discovery "popularized" by Etienne Pascal, father of Blaise Pascal (of SI-unit-for-pressure-and-"thinking reed" fame).

Investigating Supercircles and Superellipses

We've discussed equations of the form $Ax^2 + By^2 = 1$, but so far the only things we've changed are the coefficients A and B. What happens when we change the *exponent* instead? **Write up coherent answers without dangling pronouns and with neat sketches (or print them out from your calculator) to hand in next week.**

To make things more symmetrical, we'll slightly modify the original equation to

$$|x|^r + |y|^r = 1$$

with $r = 2$ corresponding to the unit circle.

✐ It should be obvious that x and y are still confined to the interval [-1,1] no matter what positive value of r we choose, but go ahead and explain why that is.

✐ If possible without using your calculator, write a formula for y in terms of x here: __________________. Then check with your calculator. (By the way, you'll need another formula too—what is it and why?)

1. Now on your calculator: Start a new problem and make a slider for r with domain [0,8], and initial value 2. Plot the functions you found above. Check that you get a ___________ , which is what you expected.

2. Change the value of r to 1. Oops! Your calculator doesn't know about the domain restriction we noticed above, so we better tell it. Select the graph of $f(x)$, go to its properties (right-click will get you there), and make it plot **discrete** points in the interval from [-1,1] – or just decide to ignore the part outside the domain. Do the same for $g(x)$.

✐ Describe your graph in words. Why aren't the sides curved?

3. Now slide r and watch the graph change. Below, make thumnail sketches of the different *kinds* of graphs you get, with approximate r-ranges for each.

4. Why does the graph do this? To answer, plot a point on the upper half of the graph, call it P, and measure its coordinates (x and y). Now watch what happens to P as you slide r.

✐ As r increases, in what direction does P move? _________ Explain algebraically. (First identify a coordinate of P that doesn't change; then describe what happens to each part of the formula as r increases, and how that changes the other coordinate of P.)

✐ Imagine the limit as $r \to \infty$: what shape would the graph have?

✐ Imagine the limit as $r \to 0$: what shape would the graph have? You can actually get the computer to produce this case for you.

✐ Do you have a favorite r-value? In particular, if you wanted to design a pond or something to fit into a roughly square space, what value of r would you choose?

Extension (worth +2 bonus)
Plot points A and B on the x- and y- axis; use **coordinates and equations** to show their coordinates, and link their x-coordinate and y-coordinate, respectively, to variables **a** and **b.** Modify your equation so that it dilates by a factor of A in the x-direction and by a factor of B in the y-direction. **NOTE:** You'll have to go back to change the function plots so they plot on a bigger domain, like $-10 \le x \le 10$. Play with these "Superellipses". What properties do they seem to share with regular ellipses? With supercircles? Write down a few observations, and give the proportions and r-value of your favorite superellipse

For Further Reading: Martin Gardner wrote a column on superellipses, a creation of Piet Hein, a Danish architect, designer, and amateur mathematician, which is reprinted as chapter 6 of Martin Gardner's *Colossal Book of Mathematics.* Piet Hein's invention isn't pure abstraction: he designed a plaza and park in Copenhagen using a "Golden Superellipse" whose length:width ratio is $\varphi = \dfrac{1+\sqrt{5}}{2}$ and $r = 2.5$.