Getting Cross with Products…

Recall from class that if $\mathbf{u} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{v} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$ the cross product $\mathbf{u} \times \mathbf{v}$ is defined as

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \mathbf{i} & \mathbf{j} & \mathbf{k} \end{vmatrix} = (x_1y_2 - x_2y_1)\mathbf{i} + (x_2z_1 - x_1z_2)\mathbf{j} + (y_1z_2 - y_2z_1)\mathbf{k}$$

The Nspire can compute dot products, cross products, and determinants. Notice that vectors are enclosed in square brackets, so $3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ is represented as `[3,1,4]`. To enter a matrix, grab a matrix from the template screen. Notice too that the Nspire is perfectly comfortable using variables in vectors and matrices.

dotP([3 1 4],[1 5 9])	44
crossP([3 1 4],[1 5 9])	[-11 -23 14]
$\det\left(\begin{bmatrix} 3 & a \\ 4 & b \end{bmatrix}\right)$	$-(4\cdot a-3\cdot b)$

Instructions: Do this assignment on your own paper. Use your Nspire judiciously: check your answers, but make sure that you can do the intermediate computations, at least in principle.

In 1 & 2, determine whether the given vectors are parallel, orthogonal, or neither.

1. < 6,-3,-9 > & < -2,1,3 >

2. < 2,-3,-4 > & < -6, -8, 3>

3. Find a vector orthogonal to both < 1,1,1> and < 2,3, -1>.

4. Find a <u>unit</u> vector orthogonal to both < 3,2,1 > and <0, -2, 5>

5. Let **i** = < 1, 0, 0 >, **j** = < 0, 1, 0> and **k** = < 0, 0, 1> . Copy and fill out the multiplication table below:

×	**i**	**j**	**k**
i			
j			
k	**k × i = -j**		

In 6-8, name the property and prove that it holds for all vectors in $\mathbf{R}^3$ (i.e. 3-D vectors).

6. $\mathbf{t} \times (r\mathbf{v}) = r(\mathbf{t} \times \mathbf{v}) = (r\mathbf{t}) \times \mathbf{v}$

7. $\mathbf{t} \times \mathbf{t} = \mathbf{0}$ (notice that this isn't the number 0!)

8. $(\mathbf{v} + \mathbf{u}) \cdot \mathbf{w} = \mathbf{v} \cdot \mathbf{w} + \mathbf{u} \cdot \mathbf{w}$

9. Prove the anticommutative Property: $\mathbf{t} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{t})$

10. Find the cosine of the angle between (1, 2, -2) and (-2, 1, 2)

11. Prove that $r\mathbf{v} \cdot \mathbf{u} = r(\mathbf{v} \cdot \mathbf{u})$.

12. Prove that $\| r\,\mathbf{v} \| = | r | \cdot \| \mathbf{v} \|$ (is the · regular multiplication or the dot product?)

13. Prove that $\| \mathbf{v} + \mathbf{t} \|^2 = \| \mathbf{v} \|^2 + 2\mathbf{v} \cdot \mathbf{t} + \| \mathbf{t} \|^2$.

14. A particle moves counterclockwise around a circle of radius 2.1 km, making one revolution every 0.3 seconds.

a. Find its angular velocity, in radians per second.

b. Find its linear velocity, in radians per second.

c. If at $t = 0$ its angle (relative to some ray from the center) is 1.1 radians, write its angle as a function of time.

15. If $\mathbf{u} = 4\mathbf{i} - 6\mathbf{j}$ and $\mathbf{v} = -10\mathbf{i} + 15\mathbf{j}$, find x and y such that $x\mathbf{u} + y\mathbf{v} = \mathbf{j}$, if such x and y exist.

16. If $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = -\mathbf{i} + 7\mathbf{j}$, find x and y such that $x\mathbf{u} + y\mathbf{v} = a\mathbf{i} + b\mathbf{j}$, in terms of a and b. What does this tell you about $\{ x\mathbf{u} + y\mathbf{v} \mid x, y \in \mathbf{R}\}$?