

No Calculators Page—When Done, Place On Overhead

Instructions: Each question is worth 8 points (split evenly among parts unless otherwise stated), no partial credit for multiple-choice problems, no full credit for short answer problems without visible work. Answers alone are worth a single point; formulas cost one point.

_____ 1. The value of $\cos \frac{\pi}{3} \cos \frac{\pi}{6} + \sin \frac{\pi}{3} \sin \frac{\pi}{6}$ is _____.

- (A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) 1

_____ 2. Suppose that $(1 + 2i)$ is a zero of the polynomial $x^3 - 3x^2 + 7x - 5$. Which of the following must be a factor of the polynomial?

- (A) $x^2 + 4$ (B) $x^2 - 5$ (C) $x^2 - 2x + 5$ (D) $x^2 + 2x + 5$ (E) $x + 1 + 2i$

_____ 3. If $z = 2(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$, then $z^4 =$ _____

- (A) -1 (B) $-8\sqrt{2} + 8\sqrt{2}i$ (C) $8i$ (D) $8\sqrt{2} + 8\sqrt{2}i$ (E) -16

_____ 4. $\csc^2 x + \sec^2 x =$ _____

- (A) $\cos^2 x \sin^2 x$ (B) 1 (C) $\csc^2 x \sec^2 x$ (D) $\cot^2 x$ (E) $\tan^2 x$

5a. Find the rectangular coordinates of a point whose polar coordinates are $(4, \frac{9\pi}{4})$.

5b. Find another set of polar coordinates for the same point.

6. A faulty thermostat sends the school's temperature into weird oscillations. The graph at right gives the temperature as a function of time. Find the value of each quantity below and briefly (< 1 sentence) say what it means in this context. (2 pts each)

a. Period: _____

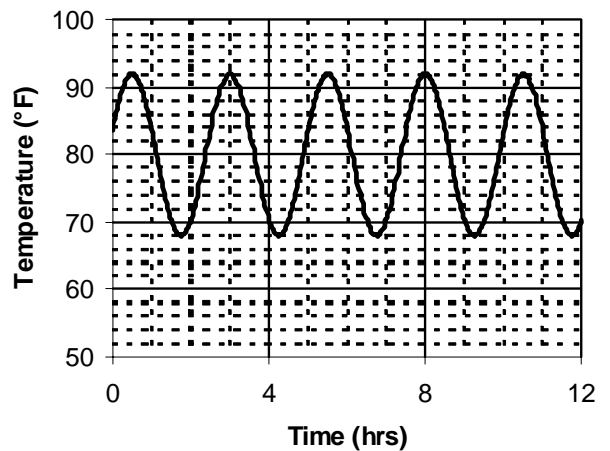
b. Amplitude: _____

c. Vertical Shift: _____

d. Phase Shift: _____

e. Formula: _____ (no explanation!)

f. IRC at $t = 5$ seconds (don't simplify beyond a fraction). _____



7. Solve for x in $[0, 2\pi)$: $\sin(4x + \pi/4) = \frac{\sqrt{3}}{2}$.

8. If $\tan x = -3/4$ and $\pi < x < 2\pi$, determine the value of $\sin 2x$.

9. Simplify: $\sec(\sin^{-1}(2x + 1))$

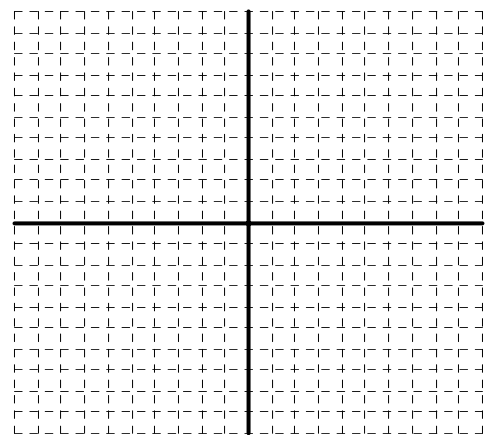
10. Consider the relation $x^2 - 4y^2 + 16y - 52 = 0$. (3 pts each part)

a. Rewrite in standard form.

b. Find the location of its foci.

c. Find the equations of its asymptotes, if any.

d. Graph the relation on the grid at right.



CALCULATORS ALLOWED

For full credit, you **MUST** show your work. Indicate what, if anything, was done on the calculator and how. Correct answers without work or logic are worth 1 point; incorrect answers without work or logic are worth 0.

- _____ 1. Justin has a 55% chance of winning any given point in a ping-pong game. To the nearest 0.1%, what is the probability that he wins exactly 7 out of the first 10 points?
- (A) 1.4% (B) 1.83% (C) 16.6% (D) 25.7% (E) 70.0%
- _____ 2. If $\mathbf{u} = 3\mathbf{i} + 8\mathbf{j}$ and $\mathbf{v} = 6\mathbf{i} - 12\mathbf{j}$, then the angle between vectors \mathbf{u} and \mathbf{v} is approximately _____ 1.
- (A) 6.01° (B) 30.1° (C) 63.4° (D) 69.4° (E) 132.9°
- _____ 3. An isosceles triangle has sides of length 5 and 12. To the nearest 0.01° , what is the measure of its base angle?
- (A) 77.98° (B) 11.77° (C) 22.62° (D) 65.36° (E) 72.3°
- _____ 4. An arithmetic sequence has $a_1 = 10$ and $a_4 = 810$. A formula for the n^{th} term of the sequence is
- (A) $10 \cdot 3^{n-1}$ (B) $10 \cdot 81^{(n-1)/3}$ (C) $10 + 200(n-1)$ (D) $10 + \frac{800}{3}(n-1)$ (E) $10n + 100n(n-1)$
5. An object is traveling around a circle of radius 14 m, at an angular velocity of $\frac{1}{4}$ radian per second.
- a. Find its linear velocity in meters per second.
- b. Find parametric equations for its path, assuming the center is at $(0,10)$ and the particle is moving clockwise, with initial angle $= 0$.
6. Nonzero vectors $\mathbf{u} = \langle \cos(x), \cos(10^\circ) \rangle$ and $\mathbf{v} = \langle \sin(10^\circ), \sin(x) \rangle$ are orthogonal. Compute all possible values of x in the interval $[0,360^\circ]$. COBRA.
7. In $\triangle ABC$, $AB = 4$ cm, $BC = 6$ cm, and $m\angle C = 12^\circ$. Find all possible values of AC (nearest 0.01).

8. Consider the series $\sum_{k=1}^{\infty} \frac{3^{2k+1}}{5 \cdot (-12)^k}$. If a_n is the n th term of the series. (10 pts)

a. Write a simplified expression for the ratio $\frac{a_{n+1}}{a_n}$. (4 pts) b. Is the series arithmetic, geometric, or neither? (2 pts)

c. Find the sum of the series. (COBRA) (4 pts)

9. Find all complex cube roots of -1. Reminder: convert to polar form first! (COBRA)

10. A parabolic dish for an antenna is to be 10 feet wide and 1 foot deep at the center.

a. How deep will it be 2 feet from the center?

b. How far away from the bottom of the dish is the focus?

11. Alyssa plans to run away to the circus, and wheels her unicycle along a tightrope suspended across the Atrium. The ends of the tightrope are 75 ft apart; as tightropes are wont to do, it sags a little. Observers at each end note angles of depression (to the place where the wheel rests on the rope) of 2.3° and 4.1° . How far down is the rope sagging? Draw a diagram, then solve the problem.

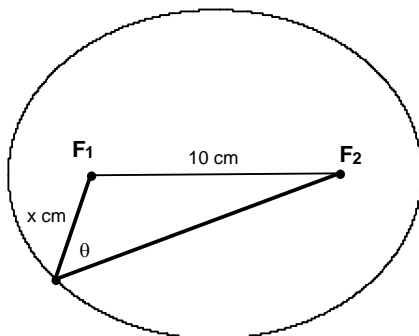
13. Leeron and Jon are flying the Goodyear Blimp over graduation ceremonies as a junior class prank. The blimp is pointed due East and flies 30 miles per hour in still air (source: <http://www.goodyearblimp.com>), but a wind is blowing at a heading of 40° at 15 miles per hour. Draw a diagram of the situation, and find their ground speed and direction.

14. PODASIP: $(\sin^2 \alpha - \cos^2 \alpha) (\tan^2 \alpha + 1) = \tan^2 \alpha - 1$.

15. A complex number z moves away from the origin along the ray $\theta = \pi/6$. Describe how each point would move (6pts)

- a. z^2
- b. A cube root of z (pick one)
- c. $1/z$

16. A 16-cm string is tied to two points 10 cm apart to make an ellipse as shown below. (3 pts each)



- a. Determine the length of the ellipse's axes.
- b. What will be the eccentricity of the ellipse?
- c. Write an expression giving θ in terms of x , the distance from point P to F_1 .

d. Find the value of x that gives the maximum value of θ .

17. Consider the points $A(4,6,2)$ and $B(-2,2,0)$.

a. Find parametric equations for the line through A and B .

b. Compute the cross product of the vectors $\overline{OA} \times \overline{OB}$
(show setup!)

18. An orange is launched at an initial velocity of 150 ft/s at an angle of 62.0° .

a. What is the maximum height it attains?

b. How far does it travel before hitting the ground again?

19. Consider the graph of $r = a + 2a \cos 3\theta$, where $a > 0$ is a real number. (4 pts each)

a. Find two values of θ for which the graph crosses the pole.

b. Find the exact polar coordinates of the points on the graph furthest from the origin.

c. How does changing a affect the graph? Explain.

20. Commuters can get to work by car or bus. Surveys have shown that for those taking their car on a particular day, 80% take their car the next day and 20% take a bus. Also, for those taking a bus on a particular day, 60% take the bus the next day and 40% take their car. In the long run what percentage of people take a bus on a particular day? Show setup.

21. To the nearest square centimeter, compute the area of a quadrilateral whose sides have lengths $AB = 200$ cm, $BC = 150$ cm, $CD = 200$ cm, $DA = 125$ cm, if $m\angle A = 65^\circ$.