Compounding Your Difficulties

After looking at your upcoming (or your friends' upcoming) tuition bills, you get scared and decide to start saving now for your children to go to college. You figure it will be about 30 years before you have college-age children (right?) and your investment advisor tells you to invest in tax-free bonds (you can withdraw the whole amount when the actual child is born and drop it into a 529, which is definitely the way to go).

✎ One year at a private university costs around $40,000. (But there is financial aid!) If expenses increase at a rate of about 5% annually (probably a little low, but bear with me) how much will it cost to go to school in 2038?

Say you find a tax-free bond promising 5% annual return. You decide to invest $2000 every year for 30 years. (For simplicity, we'll ignore the 529 rollover.) Fill in the blanks in the table below:

Deposit	Amount	Years of Interest	Value in 30 years
1	$2000	30	
2	$2000	29	
…			
N	$2000		

Now open a new document on your calculator and create a new spreadsheet page.

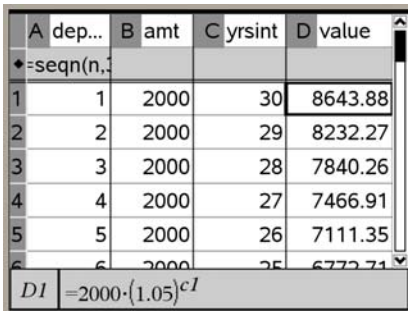
Create four columns labeled **DepNum, Amt, YrsInt, Value**, and start filling them up:

Use Menu→Actions→Data→Generate Sequence to put the numbers 1-30 in **DepNum**; fill **Amt** with the value 2000.

Let *n* be the deposit number. In terms of *n*,

✎ How many years does deposit *n* get interest? _____

✎ What is the value of deposit *n* 30 years from today? _____



Put appropriate formulas in **YrsInt** and in **Value**. Your spreadsheet should look like the one at right.

Now go to column E and call it **TotVal**. Then make the calculator compute a cumulative (partial) sum as we did last week: let E1 = D1, then in E2 put the formula **=E1+D2**. Make a scatterplot showing Totval versus DepNum.

✎ After 30 years, how much money do you have? Is it enough for four years at Whatsamatta U?

✎ Play around with the annual payment until you find a value that gives you the amount you predicted you will need for your future child's college. (Tip: you can do this quickly by simply creating a variable on a calculator screen called **pmtamt** and then using that variable in your formula in column D.)

✎ Then revert to the original $2000 payment, but change the interest rate.

Now let's do some algebra. Roll up your sleeves, fire up a calculator page, and …

✎ Let p_n be the value of the nth payment by the time you withdraw it in 2038 (i.e. the n^{th} item in column D). If p is the amount of your annual payment, i is the interest rate, write a formula for p_n in terms of p and i.

✎ The total amount you have saved in 2038 is $p_1 + p_2 + \ldots + p_{30}$. Explain why the series is geometric, and find the common ratio:

✎ Write an expression for the sum T (in terms of p and i).

$T =$

_____

✎ Now solve for p in terms of i and T. Explain in words what your new formula tells you.

✎ Quickly: your revised plan is that your child will go to U of I, where tuition, room, and board in the undergraduate business program is $28,866 and increasing at 7% annually. How much will you need for four years thirty years from now, and how much do you have to save annually at 5% to get there?

Reflection—write a paragraph or two on separate paper, and hand in with this assignment:

- Many young people don't save money early because money is tight when you're paying off loans, etc. Did your investigation change your opinion about the wisdom of this practice? (It's okay to say "no," but why?)
- How much does $2000 per year turn out to be on a weekly basis? Explain in terms of some ordinary expenditure, e.g. a trip to the movies. Did the total value after 30 years surprise you? Why or why not?

Extension:

Your parents decide to buy you a condo rather than pay for dorm space (hey, while we're daydreaming…). The cost is $150,000. Suppose that they pay 6.5% interest on that for thirty years without paying it off. How much is the value 30 years from now? _____ Now use your formula to find the annual payment—at 6.5% interest—that would equal that same amount after 30 years.

Congratulations! You've just amortized a mortgage, namely, figured out the annual payment required to pay off the loan over a given period of time. (In reality, each payment counts against the total owed, but the magic of the distributive law means that our funny way of thinking about the problem is mathematically equivalent.)

Double Extension:

Look up present value and give a reasonable discount rate: _____ . source: _____.
Now imagine that you win the lottery and they pay you $500,000 per year for 20 years. Use a similar method to the ones above to compute the present value of that series of payments.