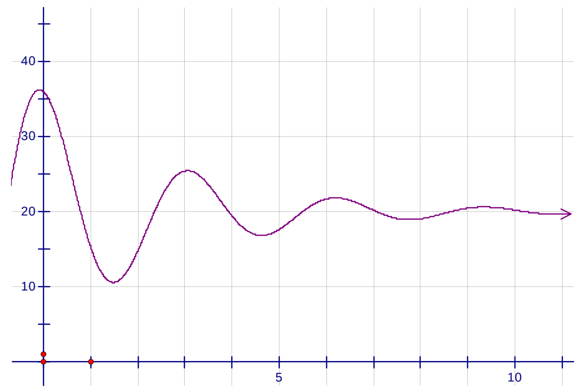
Investigating Rates on GSP

You're going to make a sketch that allows you to play with the difference quotient in two ways. The first way will show you how the values of h and $Df(x)$ relate to points on the graph; the second will allow you to manipulate h directly.

I. Roughhousing after a particularly brutal math meet, Jonathan winds up pushed out the second floor window of room 216. Luckily, his quick-thinking pal Laura snags a bungee cord over his beltloop, and he bounces up and down for a while. Jonathan's height in feet after t seconds is given by the function

$$y(t) = 2^{4-t/2} \cos(2t) + 20.$$

1. Open GSP and make a new sketch. Go to **Graph→Plot New Function** and graph $y(t)$ as given above (you'll have to use x instead of t, right?). **When it asks, switch to radians.** Then go to **Graph→Grid Form** and change to **Rectangular Grid**. Finally, adjust the scale by moving the points on the axes so that you see a nice graph of the function. Right-click on the function graph and choose **Properties** to adjust its domain to [0,some reasonably large number]. The end result should look something like the graph at right.



- In 1 or 2 sentences, describe in words the behavior of $y(t)$ starting with $t = 0$ and increasing towards $+\infty$.

2. Select the graph itself (the curve, not just the grid behind it) and select **Construct→Point on Function Plot.** Label the point A and then **Measure→*x*-coordinate** and then **Measure→*y*-coordinate**. Check that as you drag point A around on the plot, its x- and y- coordinates change.

3. Repeat step 2 except labeling the point B. Draw the *line* (not the segment) from A to B.

4. Move point A to the first relative minimum of the function and move point B to the second relative minimum. Then go to **Measure→Calculate** and compute the slope of that line using the x and y coordinates of each point that you measured before. (Remember to click on each measurement to use it in the computation.)

- What was the slope?
- Does that slope seem like a reasonable average velocity for that time interval? Is there a part of you that feels like it's weird or inappropriate? Why?

Now move point B to the first relative maximum to the right of A.

- What is the slope from A to B? Does it seem like a reasonable average?
- Is Jonathan moving at a constant velocity over the interval from A to B? If not, describe how his velocity changes over that interval.
- A theorem from calculus (or KAM!) says that between A and B there is a point C where the *instantaneous* rate of change equals the *average* rate of change from A to B. First, explain how you can tell that the intersection point of the line and the curve is *not* this magic point C.
  Then, if you can (but it's not a big deal if you can't) find the magic point C.
- Move point B closer and closer to point A. What happens to the slope? Explain.
- Find a location of point B where the slope from A to B is negative, and explain your choice.
- Now move A and B (both) to locations such that $A \neq B$ and the ARC from A to B is 0. Explain your choice.

Print Preview, Print, and go on to part II.

II. One effect of dumping pollutants in a pond can be to temporarily reduce the amount of oxygen dissolved in the water (which is required for fish to breathe). Unfortunately, nobody told Mr. Kinderman's TA, who takes a bunch of old chemicals home and dumps them in the pond at his relative (i.e. local) park. Afterwards, the amount of dissolved oxygen (expressed as a percentage of the maximum possible) is given by the function

$$O(t) = \frac{100 + 20t^2}{1 + 1.4t + 0.2t^2}$$

where t is the number of hours since the contaminants were dumped.

1. Open GSP and make a new sketch. Go to **Graph→Plot New Function** and graph $O(t)$ as given above (you'll have to use x instead of t, right?). Adjust the graph as you did in part I.

✎ In 1 or 2 sentences, describe in words the behavior of $O(t)$ starting with $t = 0$ and increasing towards $+\infty$.

2. Create a point A and measure its coordinates separately as you did in part I.

3. Now create a new parameter called **h** and make its value pretty small, like 1. Go to **Measure→Calculate** and calculate $\boldsymbol{x_A + h}$. That's the x-coordinate of the second point for your secant. To compute its y-coordinate, go to **Measure→Calculate** and calculate $\boldsymbol{O(x_A + h)}$. (Remember to click on each function or measurement to use it in the calculation.). Finally, to plot your new point, select the $\boldsymbol{x_A + h}$ calculation and the $\boldsymbol{O(x_A + h)}$ calculation in that order and select **Graph→Plot As (x,y).** Label this point B.

→ You don't have to write anything for this, but check that everything works so far: that when you move point A, point B moves also; that the horizontal distance between A and B doesn't change when you move point A; that changing the value for h changes the location of B but not A, and that everything stays on the curve.

4. Construct line $\overleftrightarrow{AB}$ and go to **Measure→Slope**. Finally, select $\boldsymbol{x_A}$ and **Slope of AB** and plot *those* as (x,y). Label the new point **M** (why?).

✎ Find a location for point A so that the line AB seems like a good approximation of a tangent line, and tell me its x-coordinate. Then find a location where the line AB seems like a bad approximation of a tangent line, and tell me its x-coordinate.

5. As we discussed, the secant is a better approximation of the tangent for smaller values of h. So choose smaller values for h until even at your "bad place" the secant seems, well, pretty tangential. We'll use that value of h for the rest of this activity. Recall that $O'(t)$ means the Instantaneous rate of change. So M's y-coordinate is simply $O'(x_A)$

✎ Slide point A until its x-coordinate is about 1. Where is M? Is $O'(t) > 0$ or < 0? How could you have predicted that just looking at the graph of $O(t)$?
✎ Repeat with A at 2, and again at 5. Which of these locations has the larger value for $O'(t)$? Again, how could you have predicted just looking at the graph of $O(t)$?
✎ Move A around until $O'(t) = 0$ (as closely as possible) and tell me A's x-coordinate. What kind of point is A?

6. To produce a graph of $O'(t)$, you only need to select point $\boldsymbol{M}$ and go to **Display→Trace Points**, then slide point A back and forth (or animate it—you'll probably want to speed things up).

✎ Describe the graph of $O'(t)$. For what values of t is it above the y-axis? Below? Rising? Falling?
✎ This is probably mean, but go ahead: for each answer in the previous pencil, try and describe one feature of the original graph of $O(t)$ that "goes with" the feature you describe in the graph of $O'(t)$. For example, if $O'(t) > 0$, what is happening on the graph of $O(t)$? Don't sweat it if you don't get a good answer for each, but give it "the college try".

Print Preview, Print, and get a puzzle from the puzzle box.