Precalculus BC Summer Assignment

Instructions: Please complete and bring to the first day of school. If you don't know how to do a problem, find out: you can look it up online (a good source is the math forum at Drexel University, mathforum.org) or ask a friend.

Use your NSpire to check answers and draw graphs but (obviously) not to do the algebra for you. If you don't have a (working) NSpire, don't despair: go to http://www.wmich.edu/cpmp/CPMP-Tools/ (because it's Java, it runs on just about any machine). The site contains a brief tutorial.

Grading: This assignment is worth 16 points (i.e. two regular, collected homework assignments).

Definition: The *Taylor Form* of the equation of a line through (x_0, y_0) with slope m is $y = y_0 + m(x - x_0)$. For example, the Taylor Form of a line through (-4,6) with slope $-½$ is $y = 6 - ½ (x + 4)$

Answers to questions 1-10 are on page 5. In 1-6, let $A = (114,6)$ and let $B = (127,1)$.

1. Find the length AB.

2. Find the equation of line $\overline{AB}$, in Taylor Form.

3. Find the equation of the line parallel to $\overleftrightarrow{AB}$ passing through (11,0). (Yes, in Taylor Form)

4. Find (a) the coordinates of point C, the midpoint of $\overline{AB}$, (b) the coordinates of point D, if A is the midpoint of $\overline{BD}$.

5. Find the coordinates of the point ¼ of the way from A to B. (How many different ways can you solve this? There are at least two.)

6. Find the coordinates of <u>at least five points</u> whose distance from A is the same as the distance from A to B.

7a. Divide $4x^2 + 7x + 11$ by $2x - 1$ and give the quotient and remainder.

7b. Now use your calculator: type **`propFrac((4x^2+7x+11)/(2x-1))`** [on CPMP tools, the command is **`expand`**] and report the answer you get. How does the output relate to your result in 7a?

7c. Now use your calculator to find the quotient and remainder when $x^4 + 4x^2 + 2$ is divided by $x - 2$.

8a. In 1985 world oil consumption was 60.1 million barrels per day. In 2000 world oil consumption was 77.0 million barrels per day. (Source: http://www.eia.doe.gov/emeu/aer/txt/ptb1110.html) Write a linear equation (yes, Taylor form) giving daily oil consumption in terms of the year, with $x = 0$ in 1980 (not 1985).

8b. Use your equation in 8a to predict the world oil consumption in 1965.

8c. The actual value in 1965 was 31.1 million barrels per day. How can you account for the discrepancy?

8d. According to some researchers, maximum daily oil production is about 100 million barrels per day. According to your equation, in what year will we reach that level of oil consumption?

9. If $f(x) = x^2 + 4x + 3$, compute

a. $f(-1)$ b. $f(0)$ c. All values of x such that $f(x) = 0$ d. $f(a - 2)$, simplified.

10. Find the coordinates of all points on the line $y = x + 2$ that are exactly four units from the point (2,1).

11. Find three solutions to the equation $(x - 4)(x^2 + 6x + 5)(x^2 + 16) = 0$.

12. If $x^3 + 5x^2 + ax + 16 = 0$ when $x = -2$, compute the value of a. Bonus for finding a factor of the polynomial!

13. Expressed in interval notation, the inequality $-6 \le x < 4$ is _____ .

(A) (-6,4) (B) (-6,4] (C) [-6,4) (D) [-6,4] (E) (-6,-4)

14. The line $4y - 2x - k = 0$ has y-intercept 2 when $k =$ _____

(A) 2 (B) 4 (C) –2 (D) 6 (E) 8

15. Solve for x: $1 + \frac{10}{x} = \frac{39}{x^2}$

16. If $3x^2 + 20x + k = 0$ has exactly one solution, then find the value of k.

17. If $4x^2y = 100$ and $z = 40x^2 + 20xy$, than find a formula for z in terms of *either* x or y alone.

18. Graphing the function $y = 6x^5 - 12x^3 + 6x + 1$ shows that the number of solutions to the equation $6x^5 - 12x^3 + 6x + 1 = 0$ is _____ .

(A) 1 (B) 2 (C) 3 (D) 5 (E) 7

19. Solve $xy - 4 = 2y + 3x$ for y in terms of x.

20. Find the coordinates of the highest point on the graph of $y = x^3 - x^2 - 5x - 2$ when

a. $0 \le x \le 2$ b. $-2 \le x \le 0$

21. What transformations to the graph of $y = \sqrt{x}$ produce the graphs of…

a. $y = \sqrt{x+4}$ b. $y = \sqrt{x} + 4$ c. $y = \sqrt{4x}$ d. $y = 2\sqrt{-x}$ (hmmm…?)

22. If the radius of a circle is increased by 4 cm, the area is increased by 48π cm^2. Find the radius of the circle.

23a. Find the domain of $\frac{x-4}{x^2 - 6x - 15}$ b. For what values of x is $\frac{x-4}{x^2 - 6x - 15} \ge 0$?

24. Simplify as much as possible: $\frac{x^2 + 5x}{x - 4} \div \frac{x^2 + x - 20}{x + 3}$.

25. Complete the square: $x^2 + 14x + 40 = (x +$ _____ $)^2 +$ ____.

26. Solve for x in terms of the ln function: $4(7^x) = 20$.

27. If $\log a = 2.1$, then find

a. $\log (a^2)$ b. $\log (10a)$ c. $\log \frac{1}{a}$ d. $\log \sqrt[3]{a}$

28. Simplify $\frac{2^{3a-2}}{(2^{a+1})^2}$. 29. Simplify to an expression with all positive exponents: $\frac{x^2y^4z^{-1}}{(xy^{-3})^2z^3}$.

30. Solve for x: $x-\sqrt{x+4}=16$.

31. If $w \neq -1$, the slope of the line between $(3w + 2, w + 5)$ and $(w, 2w + 6)$ is _____.

32. A right triangle has hypotenuse of length 15 meters. The sum of the lengths of the other two sides is 19 meters. How long is each side? Answer exactly.

33. If the graph of $y = x^2 + ax + b$ passes through (4, -2) and (-2, 4), find the values of a and b.

34. Consider the parabola $y = 6x^2 + 24x + 14$.

a. Complete the square to find the coordinates of the vertex.

b. Find the coordinates of the focus.

35. Solve each equation or inequality for x, using interval notation where appropriate.

a. $(5x - 3)(x^2 + 7x + 10)(x^2 - 68) = 0$.

b. $(11x - 2)(x + 4)(x - 3) \geq 0$

c. $(x + 3)(x - 2) \geq 50$.

d. $\frac{1}{x}+\frac{1}{x-2}=14$

36. Consider the polynomial $3x^2 + 5x - 4$, with roots p and q.

a. Compute $p + q$ b. Compute pq. c. Compute $p^2 + q^2$.

37. Write a formula for a polynomial with integer coefficients whose roots are

a. -3, 1, and 3 b. -3, 1, and 3/5 c. -3 and 1, but of degree 3, with no other real roots.

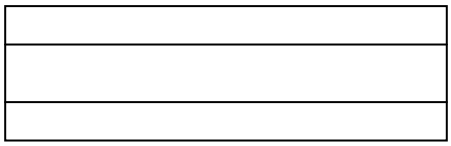
38. Find the term containing x^3 in $(\frac{1}{2}x - 4y^2)^9$ (you can use your calculator to do arithmetic).

39. Over the summer you practice shooting free throws, which you do successfully 65% of the time. What is the probability that in a series of 20 free throws you make at least 17 of them?

40. A farmer has 120 feet of fencing with which to make a rectangular pen to enclose animals.

a. Suppose he builds the pen with one side against a preexisting wall. What dimensions yield the maximum area?

b. Suppose he needs to build two dividers running the length of the pen as shown:



What dimensions yield the maximum area?

41. Factor completely:

a. $4x^2 - 24x + 4x^3$ b. $3x^3 - 24$ c. $(x + 7)(x - 2) + (x + 7)(x^2 - 8x + 8)$

42. Solve for x:

a. $x^3 \geq 4x$ b. $(x + 1)(x + 7) < x - 5$ c. $\dfrac{4}{x+2} - \dfrac{7}{x} = 0$

43. If $f(x) = 4^{x+2}$, find a value or simplified expression for each—please check on NSpire!

a. $f(-2)$ b. $f(a - 2)$ c. $f(\log_4 17)$ d. $f^{-1}(x)$ e. $\dfrac{f(x+1)}{f(x-1)}$

44. Solve each system:

a. $\begin{cases} x + y = 18 \\ 4x - 3y = 11 \end{cases}$ b. $\begin{cases} x + 2y + z = 16 \\ 2x + y + z = 18 \\ x + y + 2z = 14 \end{cases}$ c. $\begin{cases} 2^x + 4\log_3 y = 18 \\ 3 \cdot 2^x - 6\log_3 y = 45 \end{cases}$

45. Solve for x:

a. $| x + 1 | = 3$ b. $| 4x - 7 | \geq 13$

46. Give a short ($\leq$ 3 sentences) explanation, and an example of your own devising, for each statement or term:

a. The factor theorem.
b. If $| a | < 1$ and $a \neq 0$, then the equation $\sin x = a$ has <u>two</u> solutions in $[0,360°)$
c. Exponents do not distribute over addition but do distribute over multiplication.

ANSWERS to questions 1-10

1. $\sqrt{194}$

2. $y = 6 + -\frac{5}{13}(x-114)$ OR $y = 1 + -\frac{5}{13}(x-127)$

3. $y = 0 + \frac{-5}{13}(x-11)$ (yes, the 0 is optional).

4a. (120½, 7/2) b. (101,11)

5. (114 + 13/4, 6 + -5/4)

6. We'll discuss this in class.

7a. Quotient: $2x + 9/2$; remainder 31/2

7b. $\frac{31}{2(2x-1)} + 2x + 9/2$; the remainder of 31/2 appears as the numerator of a fraction whose denominator is the divisor, and the quotient appears as a polynomial added to it.

7c. You get $\frac{34}{x-2} + x^3 + 2x^2 + 8x + 16$, so the quotient must be $x^3 + 2x^2 + 8x + 16$ and the remainder must be 34.

8a. $m = \frac{77-60.1}{2000-1985} \approx 1.13$ million barrels per day per year; $y = 60.1 + 1.13(x - 5)$.
8b. 1965→x = -15, so 60.1 + 1.13(-15 – 5) = 37.5 million barrels per day.

8c. We'll discuss this one.

8d. Solve 100 = 60.1 + 1.13(x – 5) for x to get x = 40.3 or sometime in about 2020.

9a. $(-1)^2 + 4(-1) + 3 = 0$ b. 3 c. -1 is one of them (we found in part a), but factoring yields (x + 1)(x + 3), so x= -3 is the other. d. $(a-2)^2 + 4(a-2) + 3 = a^2 - 1$.

10. This is not too bad. Solve the system $y = x + 2$ and $(x-2)^2 + (y-1)^2 = 16$ for x to get $\left(\frac{\sqrt{23}+1}{2}, \frac{\sqrt{23}+5}{2}\right)$ or $\left(\frac{-\sqrt{23}+1}{2}, \frac{-\sqrt{23}+5}{2}\right)$