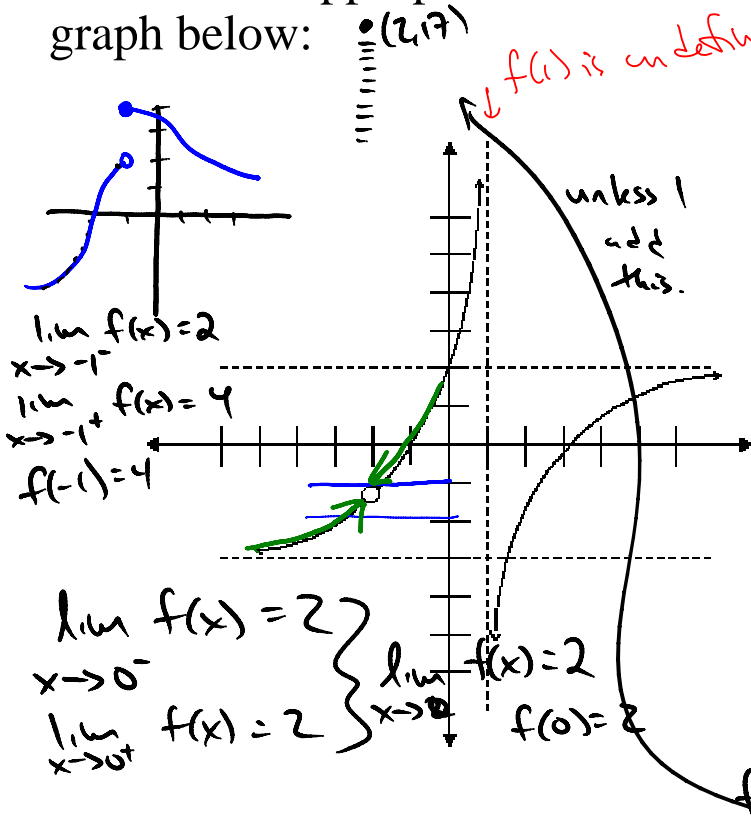


Rational Functions and Limits

Monday, December 01, 2008
9:20 AM

1. Write six appropriate limit statements for the graph below:



HAS

- ① $\lim_{x \rightarrow +\infty} f(x) = 2$
- ② $\lim_{x \rightarrow -\infty} f(x) = -3$

JAs

- ③ $\lim_{x \rightarrow 1^-} f(x) = +\infty$
- ④ $\lim_{x \rightarrow 1^+} f(x) = -\infty$

HoK

- ⑤ $\lim_{x \rightarrow -2^+} f(x) = -5/4$
- ⑥ $\lim_{x \rightarrow -2^-} f(x) = -5/4$

2. Use a calculator to estimate

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x}$$

Define $f(x) = \frac{2^x - 1}{x}$	Done
$f(0.1)$	0.717735
$f(0.01)$	0.695555
$f(0.001)$	0.693387
$f(10^{-6})$	$1000000 \cdot \left(\frac{1}{2^{1000000} - 1} \right)$
$f(10^{-6})$	0.693147
$f(0)$	undef
$\ln(2)$	0.693147
$f(-0.1)$	0.66967
$f(-0.001)$	0.692907
$f(-1.E-5)$	0.693145

Then describe what feature you would expect to find on the graph of

$$f(x) = \frac{2^x - 1}{x}$$

No VA

Hole at $(0, \ln(2))$

3. You and your friends take two cars on a skiing trip. Your friends leave at 60 miles per hour half an hour before you do. If you drive at x miles per hour, let $f(x)$ be the amount of time it takes for you to catch up. Write a formula for f , and then graph it.

$$60 \cdot \frac{1}{2} = 30$$

$$x \cdot t = 30 + 60t$$

$$x \cdot t - 60t = 30$$

$$t(x - 60) = 30$$

$$f(x) = \frac{30}{x - 60}$$

$$t = \frac{30}{x-60}$$

VA at $x=60$

HA at $y=0$

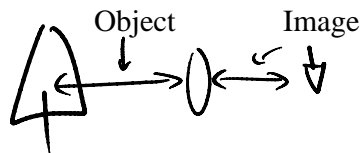
$$\left(\lim_{x \rightarrow \infty} \frac{30}{x-60} = \lim_{x \rightarrow \infty} \frac{30}{x} = 0 \right)$$

4. On the skiing trip, the fastest lift in the world takes you up at 12 m/s. You and your friends decide to compete to see who can have the fastest *round-trip* speed. If x is your speed downhill, let y be your round-trip speed. Write a formula for y in terms of x , then graph your function.

$$\frac{\text{distance total}}{\text{total time}} = \frac{2d}{\frac{d}{12} + \frac{d}{x}} = \frac{2d}{\frac{d(x+12)}{12x}} = \frac{2d \cdot 12x}{d(x+12)} = \frac{24x}{x+12}$$

$$\hookrightarrow \frac{24x}{12x} = \frac{24 \cancel{x}}{x \cdot \cancel{x} + 12 \cdot \cancel{x}} = \frac{24x}{x+12} \quad \text{HA at } y=24$$

5. You decide to take lots of photos using a camera with a fancy 200-mm focal length lens.



Use the relationship

$$\frac{1}{\text{image}} + \frac{1}{\text{object}} = \frac{1}{\text{focal length}}$$

to find out how far away the lens should be from the film when an object is x meters away.

$$\frac{1}{j} + \frac{1}{x} = \frac{1}{2} = 5$$

$$\frac{1}{j} = 5 - \frac{1}{x}, \quad j = \frac{1}{5 - \frac{1}{x}}$$

Graph your function.

$$y = \frac{x}{5x-1}$$