Inverse Functions & Rates Practice

Monday, September 22, 2008
9:15 AM

1. On Monday, September 22, the following functions would correctly perform the indicated conversions:

Conversion	Name	Formula
US$ → Euros (€)	$f(x)$	$f(x) = 1.45x$
Euros → British Pounds (£)	$g(x)$	$g(x) = 0.796x$
US$ → Brazilian Real	$h(x)$	$h(x) = 1.80x$

output / input: $100€ = 1.45x$

a. How many US$ is 100 € worth? $68.97

Use inverse to convert € to $

$f^{-1}(x) = \frac{x}{1.45}$

b. Write a formula for converting £ to €.

Is $f^{-1}(x) = \frac{1}{f(x)}$? NO

$\frac{1}{f(x)} = \frac{1}{1.45x} = [f(x)]^{-1}$

Goal: given output (y) solve for input

$y = 0.796x$

Step 1: Switch x & y

$x = 0.796y$

Step 2: Solve for y: $y = \frac{x}{0.796} = g^{-1}(x)$

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c. Describe, in words, what each function does:

i. $g \circ f$ Converts US$ → Euros → £

ii. $g \circ (h^{-1})$ h^{-1}: BR → US$

Then f: $g \circ f \circ h^{-1}$

$f^{-1} \circ g^{-1}$ → JK

ii. $g \circ (h^{-1})$ h^{-1}: BR → US$

Then f : $g \circ f \circ h^{-1}$

iii What converts £ → $ $f^{-1} \circ g^{-1}$? JK

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ Weird

2. If $f(x) = \dfrac{x}{x+5}$, find a formula for $f^{-1}(x)$.

$x = \dfrac{y}{y+5}$

$5x + xy = y$

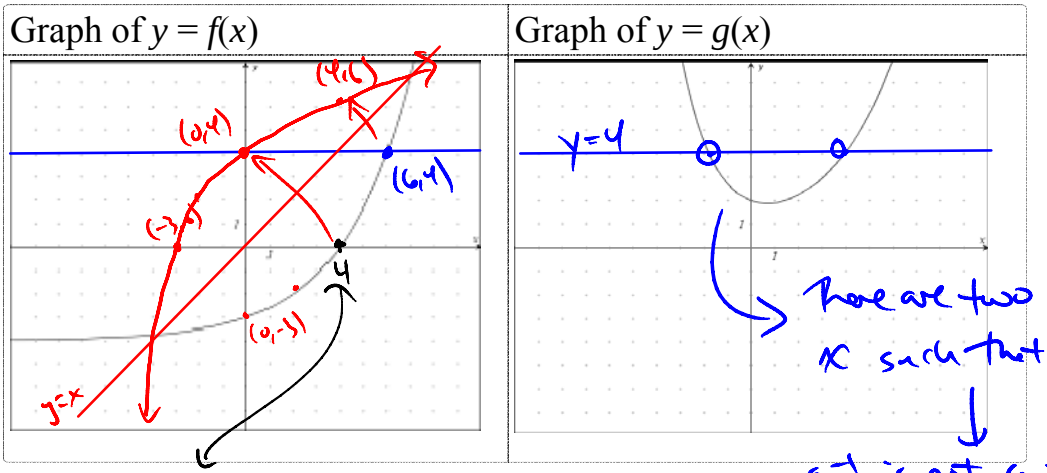
$y - xy = 5x$

$\dfrac{y(1-x)}{1-x} = \dfrac{5x}{1-x}$

$y = \dfrac{5x}{1-x}$

Calc: $y = \dfrac{-5x}{x-1}$

3. Consider the functions $f(x)$ and $g(x)$ graphed below.



f(4) = 0 Find the y-coord where x=4

a. Use the graph to estimate $f^{-1}(4)$ and $g^{-1}(4)$.

Find x when y=4 $f^{-1}(4) = 6$

There are two values of x such that g(x)=4

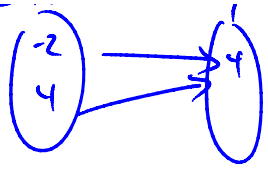
g^{-1} is not a function.

$g^{-1}(4) = 4$ or -2

g is many-to-one

b. Graph the inverse of each relation.

Horizontal Line Test: If any horizontal line $\cap$ a graph more than once, the relation's inverse is not a function.



To graph the inverse relation, switch x & y coordinates
Even: Just reflect over $y=x$

4. If $f(x)=\sqrt[3]{x+5}$, then $f(f^{-1}(x))=???$

5. If $f(x)=\sqrt{x-4}$ and $g(x)=\dfrac{1}{x^2+2}$, what is the domain of $g\circ f$?

$g(f(x)) = \dfrac{1}{(\sqrt{x-4})^2+2} = \dfrac{1}{x-4+2} = \dfrac{1}{x-2}$

Domain: $\mathbb{R}\setminus\{2\}$ or $\{x: x\neq 2\}$

~~But $x-4\geq 0$ is still required for f!~~

Moral: $\text{dom}(g\circ f) = \text{dom } f \cap \{x: f(x)\in \text{dom } g\}$

6. If $f(x)=x^2+4x$,

a. Compute the Average Rate of Change of f on [-2,2].

b. Find an expression for D$f(x)$.

c. Use your answer to (b) to compute the ARC of f on [2,2.1]