Families of Functions/Graph Terminology

Tuesday, September 09, 2008
9:54 AM

1. If $f(x) = ax^2 + 4x$, find   Substitute for every x

   a. $f(x - 2) = a(x-2)^2 + 4(x-2)$

   $= ax^2 - 4ax + 4a + 4x - 8$

   b. $f(x) - 2$

   $ax^2 + 4x - 2$

$f(3x)$ $f(x+3)$

$f(x) = ax^2 + 4x$

   c. $f(x + h)$

   d.

$$\frac{f(x+h)-f(x)}{h} = Df(x)$$

$$\frac{2axh + h^2a + 4h}{h} = \boxed{2ax + ah + 4}$$

2. Compare the domains of

$\dfrac{\sqrt{x^2+3x}}{\sqrt{x-4}}$ $\sqrt{x^2+3x}$: $x \geq 0 \cup x \leq -3$

$x(x+3) = 0 \Rightarrow x = 0$ or $x = -3$

$+ \quad - \quad +$ conc up b/c

and

$1x^2$ has pos coefficient

		-3		0	
x	$-$		$-$	0	$+$
$x+3$	$-$	0	$+$		$+$

$\sqrt{x-4}$: $x-4>0 \Rightarrow x>4$

$(x\ge 0 \cup x\le -3) \cap x>4 = x>4$

$\sqrt{\dfrac{x^2+3x}{x-4}}$

$\dfrac{x(x+3)}{x-4}$

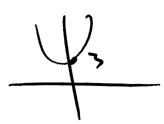
	$-$	-3	$+$	0	$-$	4	$+$
x^2+3x	$+$		$-$		$+$		$+$
$x-4$	$-$		$-$		$-$		$+$

$-3\le x\le 0 \cup x>4$

$[-3,0]\cup(4,+\infty)$

3. Find the range of $g(x) = (x^2 + 4x + 9)(x^2 + 3)$

Find range of each factor,

x^2+3  $x^2+3\ge 3$ For all $\forall x$

$x^2+4x+9 \rightarrow$ vertex @ $x=\frac{-b}{2a} = \frac{-4}{2\cdot 1} = -2$

$(-2)^2+4(-2)+9=5$

$y = (x^2+4x+9) \Rightarrow y-9=x^2+4x$

$y-9+4 = x^2+4x+4=(x+2)^2$

$y-5=(x+2)^2$

$x^2+4x+9 = (x^2+4x+\underline{4})+9-4$

$=(x+2)^2+5$

$\hookrightarrow (x^2+3)(x^2+4x+9) \geq 3 \cdot 5 = 15$

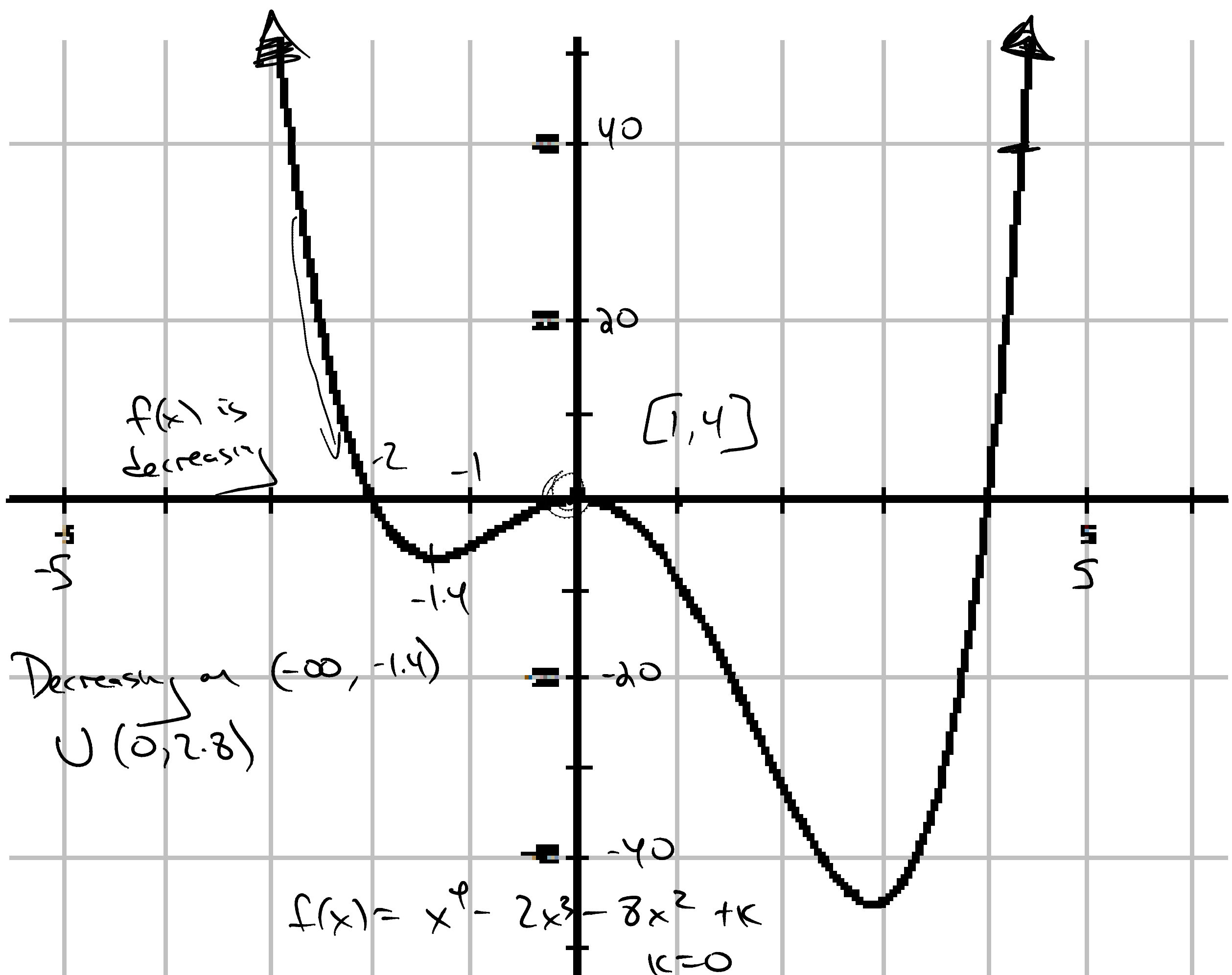
$y \geq 15$ but actually minimum value is higher

4. Consider the graph of
$f(x) = x^4 - 2x^3 - 8x^2 + k.$

$x^4 > 2x^3$ for large x

$x \cdot x \cdot x \cdot x$ $\quad 2 \cdot x \cdot x \cdot x$

The case where $k = 0$ is shown below.



Max on $[1,4] \rightarrow 0$ at $x=4$

Max on $[1,4)$ – DNE

Moral is:
To find max/min, check endpoints
If open, max/min might not exist!

a. Use the graph to solve the inequality:
$x^4 - 2x^3 - 8x^2 < 0.$ $\quad (-2,0) \cup (0,4)$
Write in interval notation.

b. Find the domain and range of $x^4 – 2x^3 – 8x^2$.

c. For what values of k does the equation $x^4 – 2x^3 – 8x^2 + k = 0$ have exactly two solutions?

On what intervals are the function's values *increasing*? *Decreasing?*

Is the point $x = 3$ the minimum value of the function? Discuss.