Precalculus BC Quiz 3-1 **Name:** Key **Per:** ___

Hungerford 8.1 & 8.5 – Intro to Identities & Inverse Trig Funcs **February 24-25, 2009**

1. Simplify as much as possible: $\dfrac{1}{\sec x - \tan x} + \dfrac{1}{\sec x + \tan x}$ (4 pts) (COBRA)

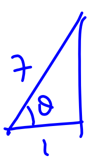
$$= \frac{(\sec x + \tan x) + (\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)} = \frac{2\sec x}{\sec^2 x - \tan^2 x}$$

Because $\tan^2 x + 1 = \sec^2 x$, this expression simplifies to

$$\frac{2\sec x}{1} = \boxed{2\sec x}$$

2. Compute (2 pts each)—for full credit, show at least one computational step

a. $\tan(\cos^{-1} 1/7)$



$\sqrt{48} = 4\sqrt{3}$

$\tan\theta = \dfrac{4\sqrt{3}}{1}$

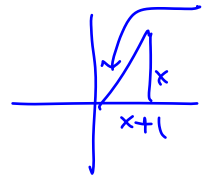
b. $\sin^{-1}(\sin(14\pi/3))$

$\sin\frac{14\pi}{3} = \frac{\sqrt{3}}{2}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

(must be in Q1 or Q4)

c. $\sec(\tan^{-1}\frac{x}{x+1})$



$\sqrt{2x^2+2x+1}$

because x could be negative, but $\sec\theta > 0$ in Q1 & Q4 (range of $\tan^{-1}$), we have $\sec\theta = \dfrac{\sqrt{2x^2+2x+1}}{|x+1|} > 0$

In 3 & 4, one equation is an identity but the other is not. Determine which is which; prove the identity and give a counterexample for the other. (Small) bonus for a salvage. (10 pts total; 2 pts each ID, 4 pts proof, 2 pts CE).

3. $\tan^4 x - 1 = \tan^2 x \sec^2 x - \sec^2 x$ ID

$\tan^4 x - 1 =$

$(\tan^2 x - 1)(\tan^2 x + 1)$

$= (\tan^2 x - 1)(\sec^2 x)$

$= \sec^2 x \cdot \tan^2 x - \sec^2 x.$

4. $(2\cos x + 3\sin x)^2 + (3\cos x - 2\sin x)^2 = 5$

Not an ID – use $x = \frac{\pi}{2}$

$(2\cdot 0 + 3\cdot 1)^2 + (3\cdot 0 - 2\cdot 1)^2 =$

$9 + 4 = 13 \neq 5.$

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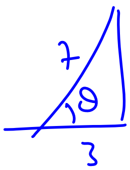
1. Simplify as much as possible: $\dfrac{1}{\csc x - \cot x} + \dfrac{1}{\csc x + \cot x}$ (4 pts) (COBRA)

$$= \frac{(\csc x + \cot x) + (\csc x - \cot x)}{(\csc x - \cot x)(\csc x + \cot x)} = \frac{2\csc x}{\csc^2 x - \cot^2 x} \quad (*)$$

but $\csc^2 x - \cot^2 x = 1$ for all x, so $(*) = \boxed{2\csc x}$

2. Compute (2 pts each)—for full credit, show at least one computational step

a. $\tan(\cos^{-1} 3/7)$



$\sqrt{40} = 2\sqrt{10}$

$\tan\theta = \dfrac{2\sqrt{10}}{3}$

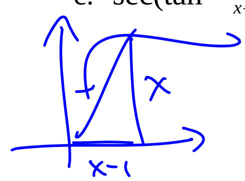
b. $\sin^{-1}(\sin(19\pi/4))$

$\sin\left(\frac{19\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

(in Q1)

c. $\sec(\tan^{-1} \frac{x}{x-1})$



$\sqrt{2x^2-2x+1}$

$-\pi/2 < \tan^{-1}\frac{x}{x-1} < \pi/2$,

so $\sec\theta > 0$; but $x-1$ could be negative, so must write $\sec\theta = \dfrac{\sqrt{2x^2-2x+1}}{|x-1|}$

In 3 & 4, one equation is an identity but the other is not. Determine which is which; prove the identity and give a counterexample for the other. (Small) bonus for a salvage. (10 pts total; 2 pts each ID, 4 pts proof, 2 pts CE).

3. $\cot^4 x - 1 = \cot^2 x \csc^2 x + \csc^2 x$

False; choose $x = \pi/2$, then $\cot x = 0$, $\csc x = 1$, and

$0^2 - 1 \neq 0 \cdot 1^2 + 1^2$

4. $(2\cos x + 5\sin x)^2 + (5\cos x - 2\sin x)^2 = 29$

True: Expand to get

$4\cos^2 x + 20\sin x\cos x + 25\sin^2 x$

$+ 25\cos^2 x - 20\sin x\cos x + 4\sin^2 x$

$= 29\cos^2 x + 29\sin^2 x$

$= 29(\cos^2 x + \sin^2 x) = 29.$

Precalculus BC Quiz 3-1 **Name: Zach Fogelson_____ Per: 6__**

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c. $\sec(\tan^{-1} \frac{x}{x-1})$

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