

**Precalculus BC  
Induction and Series Handout**

1. What combinations of postage can be made using only 5¢ stamps and 7¢ stamps? Justify your answer.
2. For what values of  $n$  can an equilateral triangle be divided into  $n$  not-necessarily-congruent equilateral triangles? Justify your answer.
3. Use algebra to prove that  $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$ . (This isn't actually that hard; write out an expression for the left hand side and start slogging through the algebra. You can write out the right hand side if it helps you figure out where you're going.)

4a. Use algebra to show that  $\frac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$ .

4b. Use your answer to part (a) to quickly find  $\sum_{i=1}^{2008} \frac{1}{\sqrt{i} + \sqrt{i+1}}$ .

5. Suppose that  $f(x) = \frac{x}{x-1}$ . Find a formula for  $f^{-1}(x)$ .

6. Find a formula for the polynomial graphed at right.

7. Solve for  $x$ :  $x(x^2 - 4) + 4(x^2 - 4) \geq 0$ . COBRA (and by "Algebra" I mean "sign graph").

8. Solve for  $x$ :  $\sqrt{2x+5} - x = 1$ .

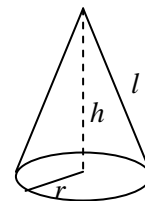
9a. Find the domain and range of  $f(x) = \sqrt{x+k}$ .

9b. Find a formula for  $f^{-1}(x)$ .

9c. Graph  $f$  and  $f^{-1}$  on the same axes. How are their graphs related?

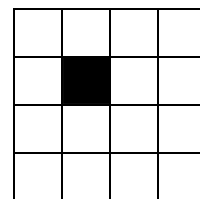
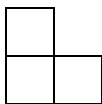
10. What transformations, in what order, produce the graph of  $y = \sqrt{4x+1}$  from  $y = \sqrt{x}$ ?

11. A cone is to have volume  $100 \text{ cm}^3$ . Find the dimensions (radius and height) that produce the smallest lateral surface area (not including the base). The formula for lateral surface area is  $A = \pi rl$ , where  $l$  is the "slant height" shown at right.



12. For what value of  $n$  is the coefficient of  $x^3$  in  $(x+1)^n$  equal to 5984?

13. Consider a  $2^n \times 2^n$  square grid with one square removed, as in the diagram at right illustrating the case  $n = 2$ . Prove that the remaining cells can be tiled (no gaps, no overlaps) with copies of the triomino below:



- First identify the base case or foundation.
- Then identify how to build from one case to the next.

