Exploring Complex Geometry with Sketchpad

Problems marked with ✋ are for you to reflect on briefly, but not to include on your final sketches. Problems marked with ✎ should be answered ON THE SKETCH with the text tool. You should PRINT OUT the versions indicated. Before printing, go to **Print Preview** and choose **Fit To Page** so that your sketches aren't cut up into some weird mosaic.

Part 0 – Getting the hang of it – nothing to hand in.

Open GSP and go to Graph→Grid Form→Square Grid.

✋ In addition to the grid itself, two points are plotted: at (0,0) and at (1,0). What does moving each point do?

✋ Plot a point **A** at (3,4) using the point tool; select the point and go to **Measure→Abscissa (*x*)** and again **Measure→Ordinate(*y*).** Now drag the (0,0) and (1,0) points around. What changes?

✋ Plot a point **B** at (3,4) using Graph→Plot Points. Drag (0,0) and (1,0) around. What changes?

Part 1 – Multiplying two numbers

Make a new sketch and put a grid on it. Plot points **Z** and **W** using the point tool (not Graph→Plot Points) and measure their *x* and *y* coordinates. Relabel the *x* and *y* coordinates of *z* to be *a* and *b* respectively, and the *x* and *y* coordinates of *w* to be *c* and *d* respectively. So $z = a + bi$ and $w = c + di$.

✋ In terms of *a, b, c, d,* write a formula for product $zw =$ _____ + (_____) *i.*

Go to Measure→Compute and enter the first part of the expression above (this will be the *x*-coordinate of the product, right?). Relabel that computation Re(*zw*). Then compute and label Im(*zw*) using the second part of the expression. Finally, select Re(*zw*), then select Im(*zw*) (so that both computations are selected), and go to **Graph→Plot as (*x*,*y*)**. Rename the point *zw*.

✎ Move point *z* to (1,0). Where does *zw* move? Move point *z* to (-1,0). Where does *zw* move?

Select point *z* and the *x*-axis, and go to **Edit→Merge Point to Axis**. Then select point *zw* and go to **Display→Trace Plotted Point**. Finally, right-click on point *z* and animate it.

✎ What shape is traced by *zw*? Why?

Select *z* and go to **Edit→Split Point from Axis.** Then merge *z* to the *y*-axis, right click on point *z*, and animate it.

✎ What shape is traced by *zw* now? Why?

PRINT OUT YOUR SKETCH. Then go to File→Document Options and copy the page so you can keep working without having to start all over again.

Select point *z* and split point *z* from the *x*-axis (Edit→Split Point from Axis). Now construct a circle centered at the origin passing through point (1,0) (remember that the control point has to BE the point (1,0)). Merge point *z* to your unit circle and repeat the experiment: erase the traces (Display menu), and animate point *z*.

✎ What shape is traced by *zw*? Be as specific as possible. Why does this happen?

✎ Don't erase the traces, but split point *z* from the unit circle and merge it to another circle centered at the origin whose radius is *not* 1. How does this change the shape is traced by *zw*? Be as specific as possible. Why does this happen?

✎ Don't erase the traces, but split point *z* from the circle and merge it to another circle not centered at the origin. Does *zw* trace the same shape as before?

PRINT OUT YOUR SKETCH. Then go to File→Document Options and make a blank page so you can keep working without having to start all over again.

Part II – Squaring complex numbers

Make a new sketch and put a grid on it. Plot point **Z** using the point tool (not **Graph→Plot Points**) and measure its *x* and *y* coordinates separately. Relabel the *x* and *y* coordinates of *z* to be *a* and *b* respectively.

- In terms of *a*, *b*, the product z^2 = _____ + (_____) *i*

As you did in Part I, compute the parts of z^2 separately, relabel, and plot the ordered pair as a point. Rename the plotted point *z*2.

- Move point *z* around until *z*2 is in each of the following locations, then describe where you had to put *z* to make that happen. (a) 1 (i.e. the point (1,0) (b) 4 (i.e. the point (4,0) (c) Any negative real number (d) Any imaginary number.

- Create a unit circle (as you did in part I) and merge point *z* to it. Where is *z*2? Animate point *z* and describe how *z*2 moves – be as specific as possible.

- Split point *z* from the unit circle and merge it to another circle centered at the origin. How does this affect your answers to the previous problem?

- Split point *z* from the circle and merge it to a line passing through the origin. What shape does z2 trace out now? Describe the shape as precisely as possible (type, location, direction, etc.)

- In a couple of sentences, use complex number arithmetic (and polar form—that's a hint!) to explain your answers to the previous questions.

PRINT OUT YOUR SKETCH.

Part III – Reciprocals of complex numbers

Make a new sketch and put a grid on it. Plot point **Z** using the point tool (not Graph→Plot Points) and measure its *x* and *y* coordinates separately. Relabel the *x* and *y* coordinates of *z* to be *a* and *b* respectively.

- In terms of *a* and *b*, 1/*z* = _____ + (_____) *i*

As you did in Part I, compute each part of 1/*z* separately, relabel them, and plot them as a point. Rename the plotted point 1/z.

- Play a little bit… as *z* moves further away from the origin, how does 1/*z* move? As *z* moves closer to the origin? If *z* moves counterclockwise around the origin? Then explain using Ang(*z*), Ang(1/*z*), | *z* |, | 1/*z* |

- Create a unit circle (as you did in part I) and merge point *z* to it. Where is 1/z? Animate point *z* and describe how 1/z moves – be as specific as possible.

- Split point *z* from the unit circle and merge it to another circle centered at the origin. How does this affect your answers to the previous problem? What about a circle *not* centered at the origin?

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- Split point *z* from the circle and merge it to a line passing through the origin. What shape does 1/z trace out now? Describe the shape as precisely as possible (type, location, direction, etc.)

- Split point *z* from the line and merge it to a line *not* passing through the origin. What shape does 1/*z* trace out now? Describe the shape as precisely as possible (type, location, direction, etc.)

- Challenge: what path does *z* need to move along in order for 1/*z* to trace out a line not through the origin?

PRINT OUT YOUR SKETCH.

The final sheaf of printed sketches is due at the beginning of next week's block.